

The deficiency spread of a transitive deficiency one parallelism of $\text{PG}(5, 2)^*$

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Partial parallelisms of $\text{PG}(n, q)$ with exactly one missing spread are known as deficiency one parallelisms. Each deficiency one parallelism can be uniquely extended to a parallelism by adding the deficiency spread. A deficiency one parallelism is transitive if it is invariant under an automorphism group that fixes the deficiency spread and is transitive on the other spreads. Transitive deficiency one parallelisms in $\text{PG}(3, q)$ are thoroughly studied in [1, 2]. The derived results are not relevant for greater dimensions $n > 3$ of $\text{PG}(n, q)$. Presently no transitive deficiency one parallelisms of $\text{PG}(5, 2)$ are known.

An automorphism of a transitive deficiency one parallelism of $\text{PG}(5, 2)$ fixes the deficiency spread, and the number of the remaining spreads is 30. Therefore the order of the full automorphism group of a possible transitive deficiency one parallelism of $\text{PG}(5, 2)$ should be divisible by 30. All parallelisms of $\text{PG}(5, 2)$ invariant under an automorphism group of order 30 and some properties of the automorphism group of a possible transitive deficiency one parallelism have been recently derived in [4], where it is proved that there are two possibilities for the deficiency spread of a transitive deficiency one parallelism of $\text{PG}(5, 2)$, namely the two unique (up to isomorphism) spreads with automorphism group orders 5760, and 362880, where the spread with the bigger group is the regular spread in $\text{PG}(5, 2)$. To search for transitive deficiency one parallelisms all subgroups of order $30m$, $m > 1$ of these two groups have to be considered. In the present work we do this for the group of order 5760.

We use the group theory functionality of GAP [3] to obtain and investigate all subgroups of order $30m$ of the group of order 5760. Then we employ our own C++ program in attempt to construct deficiency one parallelisms of $\text{PG}(5, 2)$ invariant under these subgroups. We obtain no solutions and therefore claim that if there exists a deficiency one parallelism in $\text{PG}(5, 2)$, its deficiency spread should be the regular spread of $\text{PG}(5, 2)$, and its full automorphism group should be a subgroup of order $30m$ of the full automorphism group (of order 362880) of the regular spread of $\text{PG}(5, 2)$.

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References

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